

$$\begin{aligned}
dF &= \frac{\partial G_{\sigma\nu\rho}^{000}}{\partial x^\kappa} \omega^\sigma \wedge \omega^\nu \wedge \omega^\rho \wedge \omega^\kappa + \frac{\partial G_{\sigma\kappa\rho}^{001}}{\partial \dot{x}^\nu} \omega^\sigma \wedge \omega^\kappa \wedge \dot{\omega}^\rho \wedge \dot{\omega}^\nu \\
&+ \frac{\partial G_{\kappa\sigma\rho}^{012}}{\partial \ddot{x}^\nu} \omega^\kappa \wedge \dot{\omega}^\sigma \wedge \ddot{\omega}^\rho \wedge \ddot{\omega}^\nu - \frac{\partial G_{\sigma\kappa\rho}^{112}}{\partial \ddot{x}^\nu} \dot{\omega}^\sigma \wedge \dot{\omega}^\kappa \wedge \ddot{\omega}^\rho \wedge \ddot{\omega}^\nu \\
&- \frac{\partial G_{\sigma\kappa\nu}^{012}}{\partial \ddot{x}^\rho} \omega^\sigma \wedge \dot{\omega}^\kappa \wedge \ddot{\omega}^\nu \wedge \ddot{\omega}^\rho - \frac{\partial G_{\sigma\nu\kappa}^{222}}{\partial \ddot{x}^\rho} \ddot{\omega}^\sigma \wedge \ddot{\omega}^\nu \wedge \ddot{\omega}^\kappa \wedge \ddot{\omega}^\rho
\end{aligned}$$